NASA TM- 87605

NASA Technical Memorandum 87605

NASA-TM-87605 19860015215

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OCTOBER 1985

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Space Administration

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         86N24686*#   ISSUE 15   PAGE 2414   CATEGORY 5   RPT#: NASA-TM-87605 NAS
         1.15:87605   85/10/00  19 PAGES   UNCLASSIFIED DOCUMENT
UTTL:    Structural optimization by generalized, multilevel decomposition
AUTH:    A/SOBIESZCZANSKI-SOBIESKI, J.;  B/JAMES, B. B.;  C/RILEY, M. F.
CORP:    National Aeronautics and Space Administration.  Langley Research Center,
         Hampton, Va.;  Kentron International, Inc., Hampton, Va.    AVAIL.NTIS
SAP:     HC A02/MF A01
CIO:     UNITED STATES Prepared in cooperation with Kentron International, Inc.,
         Hampton, Va.
MAJS:    /*BEAMS (SUPPORTS)/*FINITE ELEMENT METHOD/*STRUCTURAL MEMBERS/*
         SUBSTRUCTURES/*SUPPORTS
MINS:    / DYNAMIC STRUCTURAL ANALYSIS/ OPTIMIZATION/ STRUCTURAL ENGINEERING
ABA:     E.A.K.
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STRUCTURAL OPTIMIZATION BY GENERALIZED, MULTILEVEL DECOMPOSITION

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Abstract

The developments toward a general multilevel optimization capability and results for a three-level structural optimization are described. The latter is considered a major stage in the method development because the addition of more levels beyond three does not introduce any new qualitative elements, so that a three-level implementation is qualitatively equivalent to a multilevel implementation.

The method partitions a structure into a number of substructuring levels where each substructure corresponds to a subsystem in the general case of an engineering system. The method is illustrated by a portal framework that decomposes into individual beams. Each beam is a box that can be further decomposed into stiffened plates. Consequently, substructuring for this example spans three different levels: the bottom level of finite elements representing the plates, an intermediate level of beams treated as substructures, and the top level for the assembled structure. This example is an extension of a case presented previously which was limited to two levels. Further extensions would add only more intermediate substructuring levels; therefore, the three-level case is qualitatively complete.

Nomenclature

Quantities

A — Cross-sectional area

C — Cumulative constraint (equation 11)

c — Capacity: limitation on the ability to meet a particular demand D (e.g., allowable stress)

D — Demand: a physical quantity the structure is required to have, to support, or to be subjected to in order to perform its function (e.g., stress)

F — Objective function

f( ) — Functional relation

g — Vector of constraint functions, g_j

h — Equality constraints defined by equation 10

I — Cross-sectional moment of inertia

K — Stiffness matrix

KS — A function defined by equation 11

L — Lower bound on X_t including move limits

M — Mass (or "middle" when superscript)

Q — Boundary forces on SS

q — Number of the diagonal and off-diagonal symmetric entries in K

SS — A substructure (including the extremes of the assembled structure and a single structural element)

STOC — Acronym: subject to constraints

U — Upper bound on X including move limits

X — Vector of design variables, X_t

Y — Vector containing those entries of K and the mass M that are to be held constant in an SS optimization

Π — A vector defined by equation 15

ρ — A user-controlled constant in the KS function

Δ — Increment of a variable (see definition of subscript o)

Indices, Subscripts, and Superscripts

Overbar — Denotes an optimal quantity

B — Superscript for bottom level

b — Superscript to denote an association with the SS boundary

e — Subscript to identify an extrapolated value

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N86-24686

i	Index or superscript to identify the substructure level, $i = 1$ to i_{max}, see Figure 1
j	Index or superscript to identify the substructure position, counting from left, at level i, see Figure 1, or subscript of an entry in g in optimization without decomposition
k	Index or superscript to identify index j of the parent SS, see Figure 1
l	Equivalent of k, see Figure 1
M	Superscript for middle level
m	Equivalent of i, $m = i-1$, see Figure 1
n	Equivalent of i, $n = m-1$, see Figure 1
o	Subscript to identify an original value from which Δ is measured
p	Equivalent of k, see Figure 1
r	Subscript of an entry in Q, $r = 1$ to R
s	Subscript of an entry in h, $s = 1$ to S
T	As superscript denotes a transposed vector
t	Subscript of an entry in X, $t = 1$ to T
w	Subscript of an entry in g in multilevel optimization, $w = 1$ to W
z	Subscript of an entry in Y, $z = 1$ to Z

Introduction

Formal optimization methods applied to realistic structures built-up of many components and carrying a large number of loading cases are hindered by an excessive number of design variables and constraints. They frequently become too costly and unmanageable, and can easily saturate even the largest computers available today or in the foreseeable future. An obvious remedy is to break the large optimization problem into several smaller subproblems and a coordination problem formulated to preserve couplings among these subproblems. A very important benefit of such an approach, in addition to making the whole problem more tractable, is preservation of the customary organization of the design office in which many engineers work concurrently on different parts of the problem. Therefore, research has been directed toward multilevel optimization methods that decompose large problems into a hierarchically related set of smaller subproblems while preserving their coupling. This approach meshes well with the recent trend in computer technology toward computing distributed over a network of computers whose characteristics may be matched to individual subproblems for more efficiency and convenience. Moreover, the decomposition approach is natural in an engineering organization. Since engineers tend to cluster into groups concentrating on parts of a project in order to develop broad work front to shorten the development time.

A number of procedures for implementation of the foregoing approach has been proposed for structural applications, (e.g., refs. 1, 2, and 3). A multilevel optimization with decomposition has also been formulated in a general manner for use in engineering system design (ref. 4) concerned with the "trade-offs" among various physical subsystems that may be governed by different engineering disciplines. The unique feature of the algorithm proposed in reference 4 is the use of the optimum sensitivity derivatives introduced in reference 5 as means to approximate the couplings among the subsystems.

When the system optimization formulation established in reference 4 is applied to structural optimization, it's analysis part coincides with a general, multilevel substructuring (e.g., refs. 6, 7, and 8). In the simplest case, the system acquires a meaning of a complete structure and each subsystem corresponds to a single structural component that may be represented by a single finite element. This is a two-level, structural optimization whose algorithm was illustrated by an example of a framework reported in reference 9. This served as a verification of the general purpose algorithm laid out in reference 4.

Since the general algorithm presented in reference 4 allows a theoretically unlimited number of hierarchical subsystem levels in the decomposition, its continuing development requires verification by applications of more than two levels. The purpose of this paper is to report such a verification using structural optimization with a three-level decomposition. In this decomposition, the highest level corresponds to the assembled structure, the level below corresponds to substructures, and the third, bottom level, represents the structural components that make up the substructures.

Extension of the scheme beyond the three levels would require more levels of nested substructures sandwiched between the top and bottom levels. Thus, such an extension would not add any qualitatively new subsystems to the scheme and, therefore, one may regard the three-level optimization as the simplest case of the most general multilevel optimization.

The paper presents an multilevel algorithm for structural optimization and its verification by a three-level optimization of a framework structure.

One-Level Optimization

An optimization formulation without decomposition serves as a reference from which the multilevel optimization algorithm is derived.

The optimization is defined in terms of: the design variables, X_i, which are the cross-sectional dimensions of the structural components; the objective function F(X) that can be any computable function of these variables (structural mass is the frequent choice); and the constraints, $g_j(X)$, imposed on the the behavior variables to account for the potential failure modes. It is useful to distinguish here between the "local" constraints such as local buckling

that depend predominantly on the component behavior and the "global" constraints that primarily depend on the characteristics of the assembled structure (e.g., displacements or overall buckling). However, in the one-level optimization formulation both constraint categories will be treated in the same way.

Writing the constraint functions as

$$g = (D/c - 1) \quad \leq 0 \tag{1}$$

the optimization problem in a standard formulation is

$$\text{find} \quad \min_{X} F(X)$$

$$\text{STOC} \quad g_j(X) \leq 0 \tag{2}$$

and requires a search of the n-dimensional design space considering all the design variables and constraints concurrently. In contrast, an algorithm presented in the next section breaks the problem into a number of search operations, each concerned with a smaller number of design variables and constraints.

A Multilevel Optimization

Preliminary Definitions

The diagram in Figure 1 shows a structure decomposed into several levels of substructures. The term "substructure" will refer to any entity in this decomposition scheme other than the full, assembled structure represented by the box on the top of the pyramid. In the limit, then, a substructure may be a single structural component representing the ultimate geometrical detail appropriate to the problem at hand. The substructure levels are numbered from 1 on the top to imax at the bottom. The hierarchical nature of the scheme instigates the use of a term "parent" to the structure at level i which, in turn, is decomposed into a number of "daughter" substructures at level i+1. A daughter may have only one parent and that parent must be at the level immediately above. Thus, it will be convenient to label each substructure SSijk, where i denotes the level, j defines the position at the level i counting from the left, and k identifies the parent's position at the level i-1. The substructure occupying the last position in a particular parent-daughter succession represent the ultimate level of detail at which the decomposition stops. There is no requirement that all such structures must be at the same bottom level imax. Any particular parent-daughter succession line may end at $1 < i <=$ imax.

Substructuring analysis (e.g., refs. 6, 7, 8) of a parent substructure SSijk, yields the forces acting on the boundary of each daughter substructure. In the general case these forces, collected in a vector Q, depend on the stiffness properties of all the daughter substructures so that

$$Q = f(K^{bi+1,j}) \tag{3}$$

where $K^{bi+1,j}$ denotes the boundary stiffness matrix of the jth daughter substructure.

The stiffness matrix K^{ij} of the SSijk is assembled of the matrices $K^{bi+1,j}$ - the fact symbolically expressed as

$$K^{ij} = S(K^{bi+1,j}) \tag{4}$$

where S stands for an appropriate stiffness summation operator.

Similarly, the mass, M^{ij}, of SSijk is a simple sum of the daughter masses

$$M^{ij} = \sum_{j} M^{i+1,j} \tag{5}$$

For a substructure SSijk that is at the ultimate level of detail and, therefore, is not further subdivided, the stiffness matrix, K^{bij}, and the mass, M^{ij}, derive directly from the cross-sectional dimensions denoted by X_t^{ij} so that

$$K^{bij} \equiv K^{ij} = f(X_t^{ij}) \tag{6}$$

and

$$M^{ij} = f(X_t^{ij}) \tag{7}$$

The same cross-sectional dimensions and other appropriate geometrical and material data can be entered together with the forces Q into a procedure to calculate stresses, internal forces, and the critical stresses (and/or internal forces) for local buckling. This information, together with the displacements calculated in the analysis of the entire structure and each substructure and overall elastic stability analysis results, describe the static elastic behavior that may be subjected to constraints in the optimization.

Although the foregoing definition of substructuring analysis is based on the stiffness approach, the use of a finite element analysis is not mandatory for the multilevel optimization algorithm whose description will follow. As far as that algorithm is concerned, the analysis is a "black box" where only the inputs and outputs are important but not the content.

Multilevel Optimization Algorithm

With the substructuring scheme and analysis established in the foregoing, this section describes the optimization algorithm itself. The essentials of the computer implementation are also given.

Basic Concept

The basic concept of the multilevel structural optimization introduced in reference 9 is based on the well-known property of a substructure that the elements of the boundary stiffness matrices of its daughters on the right-hand side in equation 4 can be changed in such a way that

its stiffness matrix, K^{bij}, will be held constant and, therefore, the boundary forces, Q, will remain unchanged. In the case of a substructure that is not a parent, the same applies to its cross-sectional dimensions and its stiffness matrix, K^{ij}, provided, of course, that the number of symmetric entries, q^{ij}, in K^{ij} is smaller than the number of cross-sectional dimensions, T^{ij}, in the vector X^{ij}.

$$q^{ij} < T^{ij} \tag{8}$$

When this inequality holds, there is a design freedom to "tailor" the substructure to a set of prescribed stiffness properties. Otherwise, if

$$q^{ij} \geq T^{ij} \tag{9}$$

then the set of prescribed stiffness properties either defines the cross-sectional dimensions uniquely or cannot be physically realized.

Based on the property discussed in the foregoing, the optimization scheme introduced in reference 9 and extended here uses the elements of the boundary stiffness matrices, $K^{bi+1,j}$, of the daughters as design variables of their parent substructure. That definition of the design variables applies to all substructures except those that have no daughters. Their design variables are their cross-sectional dimensions.

Optimization At The Most Detailed Level

Introduction of the optimization algorithm begins at the level of the most detailed substructures that are not further subdivided. It is assumed that a complete, top-down, substructuring analysis has been carried out so that for an SSijk one has computed its boundary forces Q^{ij}. The prerequisite completion of the analysis implies that the entire structure has been initialized. Consequently, the SSijk has its mass, M^{ij}, cross-sectional dimensions, X_t^{ij}, and the entries of its stiffness matrix, K^{ij}, as the given quantities.

The optimization problem to be solved for SSijk calls for the cross-sectional dimensions to be treated as design variables and manipulated so as to minimize a measure of the constraint violations local to SSijk while holding the elements of matrix K^{ij} and the mass M^{ij} constant and staying within the side constraints.

To formalize this, define:

X^{ij} — Vector of the design variables, X_t^{ij}, selected among the cross-sectional dimensions; $t = 1$ through T^{ij}.

Y^{ij} — Vector containing the entries, Y_z^{ij}, of the stiffness matrix K^{ij} and the mass M^{ij} to be held constant; $z = 1$ through Z^{ij}.

Q^{ij} — Vector of the boundary forces, Q_r^{ij}; $r = 1$ through R^{ij}.

g^{ij} — Vector of the inequality constraint functions, g_w^{ij}, $w = 1$ through W^{ij}.

h^{ij} — Vector of the equality constraint functions, h_s^{ij}; $s = 1$ through S^{ij}. These constraints link Y^{ij} and X^{ij} so that considering equation 6

$$h^{ij} = Y^{ij} - f(K^{bij}) = 0 \tag{10a}$$

$$h^{ij} = Y^{ij} - f(X^{ij}) = 0 \tag{10b}$$

where f( ) denotes a known, computable function.

L^{ij}, U^{ij} — Vectors of lower and upper limits on the design variables X_t^{ij}.

C^{ij} — Cumulative constraint, a single valued function of g^{ij}, continuous and differentiable, having the property of being positive when at least one constraint g_w^{ij} is positive (that is violated in the convention adopted here). Following reference 9 the cumulative constraint is chosen in form of the Kresselmeir-Steinhauser (KS) function (ref. 10).

$$C^{ij} \equiv KS(g_w^{ij}) = \frac{1}{\rho} \ln \left[\sum \exp\left(\rho g_w^{ij}\right)\right] \tag{11}$$

that has the property of approximating the maximum constraint so that

$$\mathrm{MAX}(g_w) < KS < \mathrm{MAX}(g_w) + \frac{1}{\rho}\ln(W) \tag{12}$$

with the factor ρ controlled by the user. Thus, the KS function serves as a convenient single measure of the degree of constraint violation (or satisfaction).

In addition to the above functions the following functional relationship exist:

$$g^{ij} = f(X^{ij}, Y^{ij}, Q^{ij}) \tag{13}$$

The optimization problem definition is

$$\min_{X^{ij}} \quad C^{ij}(X^{ij}, Y^{ij}, Q^{ij}) \tag{14a}$$

$$h^{ij} = 0 \tag{b}$$

$$L^{ij} \leq X^{ij} \leq U^{ij} \tag{c}$$

In this problem, Y^{ij} is being held constant by virtue of equation 14b. Consequently, Q^{ij} remain constant also and together with Y^{ij} form a set of parameters of the optimization problem.

Solution of this optimization problem (by any technique available) yields a constrained optimum described by a vector π^{ij} composed of the minimum value of the cumulative constraint, $\bar{C}^{ij}$, and the optimal vector of the design variables, $\bar{X}^{ij}$

$$\pi^{ijT} = [\bar{C}^{ij} \vdots \bar{X}^{ij}]^T \tag{15}$$

This solution is sensitive to the parameters of the problem, Q^{ij} and Y^{ij}. That sensitivity, which will be used to approximate the daughter-parent coupling, may be quantified, as in reference 9 by means of the optimum sensitivity derivatives (ref. 5). Considering that Q^{ij} depends on Y^{ij} (equation 3 and the definition of X^{ij}), the total derivative of $\bar{C}^{ij}$ with respect to Y_z^{ij} is

$$\frac{d\bar{C}^{ij}}{dY_z^{ij}} = \frac{\partial\bar{C}^{ij}}{\partial Y_z^{ij}} + \sum_r \frac{\partial\bar{C}^{ij}}{\partial Q_r^{ij}} \frac{\partial Q_r^{ij}}{\partial Y_z^{ij}} \tag{16}$$

In equation 16 the partials of $\bar{C}^{ij}$ with respect to Y_z^{ij} and with respect to Q_r^{ij} are obtained from the algorithm described in reference 5, and the partial Q_r^{ij} with respect to Y_z^{ij} can be calculated by conventional structural sensitivity analysis. Similarly,

$$\frac{d\bar{X}^{ij}}{dY_z^{ij}} = \frac{\partial\bar{X}^{ij}}{\partial Y_z^{ij}} + \sum_r \frac{\partial\bar{X}^{ij}}{\partial Q_r^{ij}} \frac{\partial Q_r^{ij}}{\partial Y_z^{ij}} \tag{17}$$

Optimization Of A Parent Substructure

As shown in figure 2, the parent substructure SSmkl, m = i-1, receives from its daughters, SSijk, the minimized values of their cumulative constraints, $\bar{C}^{ij}$, optimal values of their design variables, $\bar{X}^{ij}$, and the optimum sensitivity derivatives of these quantities with respect to parameters, Q^{ij} and Y^{ij}, according to equations 15 through 16. The substructure SSmkl itself is a daughter of SSnlp, n = m - 1, that acts on SSmkl with the boundary forces, Q^{mk}, and governs its boundary stiffness matrix by means of the parameters Y^{mk}.

The design freedom in SSmkl consists in the freedom to manipulate the stiffness and mass distributions among its daughters by means of the design variables X_t^{mk} collected in a vector X^{mk}. That vector includes as partitions all the vectors Y^{ij} whose entries were parameters in optimizations of the daughters SSijk.

The optimization problem in SSmkl is basically the same as the one formulated previously for each daughter SSijk. It calls for finding a vector X^{mk} that minimizes a cumulative constraint, C^{mk}, for SSmkl. That constraint includes the constraints g^{mk} representing the limits that may be imposed in the SSmkl own behavior, e.g., the interior displacements. It also includes the quantities $\bar{C}^{ij}$ to account for the changes in $\bar{C}^{ij}$ caused by the variability of the X^{mk}. Ordinarily, every variation in X^{mk} would require a reoptimization of the affected daughters SSijk in order to find the new values of $\bar{C}^{ij}$ and $\bar{X}^{ij}$. However, following the concept introduced in reference 4 and applied in reference 9 these new values will be approximated by a linear extrapolation using the optimum sensitivity derivatives. Thus, the potentially costly reoptimizations are bypassed.

Formalization of the above optimization problem follows the pattern established for the daughter substructure SSijk.

Definitions:

X^{mk} — Vector of the design variables, X_t^{mk}, t = 1 through T^{mk}, related to the vectors Y^{ij} by

$$X^{mkT} = [\ \dots \vdots Y^{ij} \vdots \dots\ ]^T \tag{18}$$

Y^{mk} — Vector containing the entries, Y_z^{mk}, z = 1 through Z^{mk}, of the boundary stiffness matrix, K^{bmk}, and the mass, M^{mk}. These quantities are to be held constant in the ensuing optimization and in order to formulate an appropriate equality constraint one needs to recognize the functional relationships discussed below.

The matrix K^{bmk} is a function of the stiffness matrix K^{mk}

$$K^{bmk} = f(K^{mk}) \tag{19}$$

This function is no longer a simple identity as it was in equation 6, but represents elimination of the interior degrees of freedom by means of a solution of a set of simultaneous linear equations with many right-hand sides (e.g., refs. 6, 7, 8).

In equation 19, the matrix K^{mk} in turn is a function of the entries of the matrices K^{ij} through a stiffness summation algorithm

$$K^{mk} = S(K^{ij}) \tag{20}$$

Hence, by equation 6

$$K^{mk} = f(X^{ij}) \tag{21}$$

Following equations 5 and 7, the mass, M^{mk}, is

$$M^{mk} = f(X^{ij}) = \sum_j M^{ij} \tag{22}$$

Q^{mk} — Vector of the boundary forces, Q_r^{mk}; r = 1 through R^{mk}.

g^{mk} — Vector of the inequality constraint functions, g_w^{mk}; w = 1 through W^{mk}.

h^{mk} — Vector of the equality constraint functions, h_s^{mk}; s = 1 through S^{mk}. These constraints link Y^{mk} and X^{mk} so that, considering equations 18 through 22.

$$h^{mk} = Y^{mk} - f(K^b) = 0 \tag{23}$$

and, hence

$$h^{mk} = Y^{mk} - f(X^{mk}) = 0 \tag{24}$$

L^{mk}, U^{mk} — Vectors of lower and upper limits on the design variables X_t^{mk}. In this application, the L^{mk} limits are needed to keep the diagonal entries of K^{ij} and the masses M^{ij} as nonzero, positive values, and also to prevent the off-diagonal entries of K^{ij} from assuming physically unrealizable values.

Another important role of the upper and lower limits is to represent the move limits needed to prevent excessive errors in the linear extrapolation of $\bar{C}^{ij}$ and $\bar{X}^{ij}$.

$\bar{C}_e$ — Vector of the daughter cumulative constraints, $\bar{C}^{ij}$, estimated by a linear extrapolation and included in the cumulative constraint C^{mk}. The extrapolation is accomplished by a linear portion of the Taylor series using equation 16 taking into account that the X_t^{mk} replaces the Y_z^{ij} by virtue of the definition in equation 18, so that:

$$\bar{C}_e^{ij} = \bar{C}^{ij} + \sum_t \frac{d\bar{C}^{ij}}{dX_t^{mk}} \Delta X^{mk} \tag{25}$$

C^{mk} — A cumulative constraint for SSmkl representing all functions in the vectors g^{mk} and $\bar{C}_e$ through the KS function introduced in equation 11, so that

$$C^{mk} = 1/\rho \ln \left[\sum_w \exp(\rho g_w) + \sum_{ij} \exp(\rho C_e^{ij})\right] \tag{26}$$

Finally, owing to equation 24 there are functional relationships analogous to equation 13 extended to include the cumulative constraints $\bar{C}_e^{ij}$

$$g^{mk} = f(X^{mk}, Y^{mk}, Q^{mk}) \tag{27}$$

$$\bar{C}_e^{ij} = f(X^{mk}, Y^{mk}, Q^{mk}) \tag{28}$$

Based on the above definitions, the optimization problem for the substructure SSmkl is

(29)

$$\min_{X^{mk}} C^{mk}(X^{mk}, Y^{mk}, Q^{mk}) \quad \text{a)}$$

$$h^{mk} = 0 \quad \text{b)}$$

$$L^{mk} \le X^{mk} \le U^{mk} \quad \text{c)}$$

$$L^{ij} \le \bar{X}_e^{ij} \le U^{ij} \quad \text{d)}$$

where

$$\bar{X}_e^{ij} = \bar{X}^{ij} + \frac{d\bar{X}^{ij}}{dX_t^{mk}} \Delta X^{mk} \tag{30}$$

The increment $\Delta \bar{X}_e^{ij}$ is

$$\Delta X^{mk} = X^{mk} - X_0{}^{mk} \tag{31}$$

due to equation 30 in which X_t^{mk} stands for Y_z^{ij} because of equation 18.

The constraints expressed by equation 30 are introduced to reduce the probability that the optimization of SSmkl might induce overstepping of the side constraints in the daughters SSijk. However, evaluating these constraints for the quantities approximated by equation 30 does not guarantee that such overstepping will not occur. Other means are needed to prevent that as it will be explained in the discussion of the entire iterative procedure. It follows that the constraints of equation 29d are not essential and may be omitted. Indeed, there is a strong motivation to omit them because to do so would allow limiting the optimum sensitivity analysis to calculation of the objective function derivatives. This according to reference 5, requires an input of only the behavior gradients while calculations of the optimal design variable derivatives requires input of the gradients and the second derivatives of behavior - a very substantial different in the computational cost.

In view of the above, a distinction will be made between variant 1 and the algorithm that includes equation 29d and variant 2 in which it is omitted. For completion of the presentation, the algorithm description will continue for variant 1.

The SSmkl optimization produces a constrained optimum described by a vector Π^{mk} composed of the minimum value of the cumulative constraint, $\bar{C}^{mk}$, and the optimal vector of the design variables, $\bar{X}^{mk}$.

$$\Pi^{mkT} = [\bar{C}^{mk} \vdots \bar{X}^{mk}]^T \tag{32}$$

It is followed by the analysis of sensitivity with respect to parameters Y^{mk} and Q^{mk} considering that the forces Q^{mk} depend on Y^{mk} through analysis of SSmkl. The optimum sensitivity derivatives are

$$\frac{d\bar{C}^{mk}}{dY_z^{mk}} = \frac{\partial\bar{C}^{mk}}{\partial Y_z^{mk}} + \sum_r \frac{\partial\bar{C}^{mk}}{\partial Q_r^{mk}} \frac{\partial Q_r^{mk}}{\partial Y_z^{mk}} \tag{33}$$

$$\frac{d\bar{X}^{mk}}{dY_z^{mk}} = \frac{\partial\bar{X}^{mk}}{\partial Y_z^{mk}} + \sum_r \frac{\partial\bar{X}^{mk}}{\partial Q_r^{mk}} \frac{\partial Q_r^{mk}}{\partial Y_z^{mk}} \tag{34}$$

which by virtue of recursivity of the decomposition are identical to equations 16 and 17, except the indexes ij being replaced by mk.

The data defined by equations 32, 33 and 34 are carried from SSmkl to its parent SSnlp at the level $n = m-1$.

Optimization Of The Next Parent Structure

Moving on to the substructure SSnlp, everything that was stated in the preceding subsection on optimization of SSmkl applies to SSnlp literally, provided that: the indexes n, l, and p are replaced by another triplet, say, a, p, c, that identifies the parent of SSnlp at the level $a = n-1$; and the indexes m, k, and l are replaced by n, l, and p. For generality of variant 1, one needs also to extend equation 30 to encompass fully each line of succession emanating downward from SSnlp. Beyond these changes, no new conceptual elements are introduced, and no additional definitions or discussion are needed at the junctions between the levels until one arrives at the top level. Hence, any number of intermediate levels of substructuring can be inserted, if physically justified, into a line of succession connecting the assembled structure on the top to any most detailed substructures below. This property characterizes the algorithm as recursive.

Optimization At The Highest Level

The assembled structure is designated SS110. Its optimization problem is similar to the one described for SSmkl with the following differences:

1. No parameters are defined solely for the decomposition purposes.
2. The objective function is the mass of the assembled structure.
3. There is no need for a cumulative constraint although there still is an option to use it to reduce the number of constraints that need to be processed.
4. The boundary forces are the external loads on the assembled structure.
5. There is no need for the equality constraints to enforce constancy of the mass and the boundary stiffness entries.

The definition needed to formulate the top level optimization problem are identical to those given for SSmkl, omitting those that do not apply because of the differences 1 through 5 above. The remaining definitions are:

X^{11} Vector of the design variables, X_t^{11}, t = 1 through T^{2k}, related to the vectors Y^{2j} by

$$X^{11T} = [\ \dots \vdots Y^{2j} \vdots \dots\ ]^T \tag{35}$$

The mass, M^{11} is

$$M^{11} = f(X^{11}) = \sum_j M^{2j} \tag{36}$$

Q^{11} Vector of the external loads, Q_r^{11}, reduced to the boundary degrees of freedom according to the conventional formulation of substructuring; r = 1 through R^{11}.

g^{11} Vector of the inequality constraint functions, g_w^{11}, imposed on the displacements, buckling critical loads, or internal forces in SS110; w = 1 through W^{11}.

L^{11}, U^{11} Definition for L^{mk}, U^{mk} that followed equation 24 applies with the indexes ij replaced by 2j, and the indexes mk replaced by 11.

$\bar{C}_e$ Definition given in conjunction with equation 25 applies with the index replacements as above.

Finally, based on the above definitions, the optimization problem at the top level is

$$\min_{X^{11}} M^{11}(X^{11}) \quad \text{a)} \tag{37}$$

$$g^{11} \le 0 \quad \text{b)}$$

$$\bar{C}_e^{2j} \le 0 \quad \text{c)}$$

$$L^{11} \le X^{11} \le U^{11} \quad \text{d)}$$

$$L^{2j} \le \bar{X}_e^{2j} \le U^{2j} \quad \text{e)}$$

where equation 37e is analogous to equation 29d with the limits L^{2j}, U^{2j} reflecting the limits passed upwards through extrapolations of the type expressed by equation 30 extended recursively to encompass all the levels below as mentioned in the subsection on SSnlp.

Unlike all the daughters SSijk, the optimization of SS110 does not have to be analysed for the optimum sensitivity.

Iterative Procedure

When the SS110 optimization is completed, the entire structure has acquired a new distribution of stiffness and mass within the move limits. Hence, the analysis must be repeated and followed by a new round of substructure optimizations in an iterative manner until convergence. The iterative procedure is composed of the following steps:

1. Initialize all cross-sectional dimensions.

2. Perform a substructuring analysis, including for each substructure at each level the transformation of the stiffness matrix into the boundary stiffness matrix, and the transformation of the forces applied to the interior degrees of freedom to the forces coinciding with the boundary degrees of freedom.

   Calculations of the behavior derivatives needed for the ensuing optimizations and for the optimum sensitivity analyses are implied in the substructuring analysis.

3. Perform the operations of optimization and optimum sensitivity analysis as defined by equations 10 through 34.

4. Optimize the assembled structure as defined by equation 35 through 37.

5. Repeat step 2 and terminate when: all constraints g^{ij} are satisfied at all levels, and M^{11} has entered a phase of diminishing returns. Otherwise, continue.

Salient Features Of The Algorithm

Taking a perspective view on the multilevel algorithm with the algorithm for optimization without decomposition as a reference, the following salient features stand out:

1. Optimization by decomposition replaces optimization of a single large problem with optimizations of a multitude of smaller problems which are isolated from each other. Obviously, all the subproblems at a given level can be analyzed and optimized simultaneously.

2. The design variables above the most detailed substructures are generalized design variables that enable the designer to control the structure behavior by controlling its stiffness and mass distributions.

3. The structure mass is controlled at the assembled structure level. The optimizations at the lower levels are concerned with improving the constraint satisfaction.

4. The coupling among the subproblems is approximated by estimating the changes in the substructure behavior caused by the changes in the higher level substructures by means of linear extrapolations. In this respect, the approach resembles the piecewise linearization technique known to be effective in many optimization application (e.g., ref. 3).

5. The multilevel and corresponding single level optimizations can be regarded as equivalent in the sense discussed in (ref. 9, App. B). Although no rigorous, optimality-condition based proof has as yet been produced, the equivalence assertion is supported by all the application experience to date. The equivalence means that a multilevel optimization and a single level optimization of the same problem will arrive at the same solution, if the problem is convex. In a non-convex problem, different solutions are likely because the two algorithms are different and can follow different search paths in the design space.

6. From the view point of the overall optimization procedure, the operations of substructure analysis, optimization, and optimum sensitivity analysis are "black boxes" whose content can be freely replaced as long as their input/output remain as defined in the foregoing. Dissimilar algorithms may be used for the same operation applied to different substructures. In particular, although one would expect a finite element method to be used throughout for substructuring analysis, it is not a requirement as illustrated by the numerical example in the next section.

7. The design freedom (equation 8 and 9) must exist between each parent and daughter substructures. Normally, that freedom is assured, if none of the daughter substructures is of the ultimate detail type. This is because the lack of such freedom would be an indication that the daughters do not contribute enough stiffness entries to the parent to support all of its degrees of freedom - a sign of incorrect substructuring. Given a design freedom, there still is the issue of the completeness of the control a designer wants to exert over the stiffness and mass distribution in a substructure. That control may be complete, if all entries of the substructure boundary stiffness matrix and the mass of a daughter substructure are design variables in the parent substructure. The control may be incomplete if one decides to manipulate as design variables only a subset of these quantities leaving the remainder free to float. The degree of control that one needs is probably problem-dependent and involves engineering judgment.

Intuitively, it appears likely that in many applications an incomplete control, e.g., control over only the diagonal entries of the boundary stiffness matrix and the mass, should be adequate. In some cases, a complete control may not be possible physically, e.g., one can not control all stiffnesses of a generally anisotropic composite material plate, if there are not enough plies in its layup. The completeness of the stiffness and mass control remains to be a research issue to be investigated further.

Numerical Example

The algorithm presented in the preceding sections was tested on the portal framework structure that also served as a test case in reference 9. The structure is illustrated in Figure 3. In the original construction used in reference 9 the three beams of the framework had a thin-walled, I-shaped cross-section, therefore, the structure decomposed into two levels: assembled framework and individual beams. Figure 3 shows how the construction was modified by replacing the I-cross-section beams with box beams. Each box beam is built-up of four thin, stringer reinforced walls in order to provide a third decomposition level. Although the structure appears simple, the experience that has accumulated since it was introduced in reference 9 shows (references 11 and 12) that it is a challenging optimization test replete with local minima.

Testing of the algorithm was carried out by first optimizing the structure without decomposition in a conventional manner to establish a reference and then optimizing it as a three-level system. Each multilevel optimization was started from different points in the design space and the results were evaluated against the reference results.

Description Of The Test Problem

The construction, loading, design variables, constraints and the objective function of the test structure are:

1. Construction: As shown in Figure 3. The box beams are symmetric with respect to the plane of the figure. The material is an Al-alloy with properties given in Appendix.

2. Loading: Figure 3 shows the loads applied to the structure. The concentrated force and the moment constitute two, independent loading cases.

3. The design variables are the cross-sectional dimensions labeled X_1^B through X_6^B in Figure 3, DETAIL B and X_1^M through X_5^M in Section AA. The thicknesses 1, 3, and 4 in a box beam are equivalent wall thicknesses and incorporate the "smeared" stringers so that the thickness of the sheet metal itself is not an independent variable but results from the stringer dimensions and the equivalent wall thickness. The total number of the independent design variables is, considering symmetry of the beam cross-section: $6 * 3 * 3 + 2 * 3 = 60$. The constraints are evaluated using the analysis tools described in the next subsection.

4. Constraints: Horizontal translation and rotation at the loaded corner are limited to .25 in and 0.005 rd, respectively. The beams must not buckle in a Euler column mode. The stiffened plates in the box beams must not develop stresses higher than the allowables stated for the material, and they must not fail by local buckling. There are also minimum gage constraints on the thicknesses and side constraints on the other cross-sectional dimensions. The cross-sectional geometrical proportions are also restricted by such obvious considerations as the need to keep each stringer from protruding too far into the interior of the box beam. The total number of constraints was 265.

5. The structural material volume is the objective to be minimized.

Analysis and Design Space Search Tools

Calculation of displacements of the framework joints and the end-forces acting on its three beams was carried out using a small finite element program based on the displacement method. Each beam was modeled as a single beam element characterized by its cross-sectional area and bending moment of inertia in the framework's plane. In this phase of analysis the problem was treated as two-dimensional and each nodal point had only three elastic degrees of freedom. The support points were assumed clamped. The program was capable of calculating analytically the static behavior first and second order sensitivity derivatives with respect to the cross-sectional properties of area and moment of the inertia.

Analysis of an individual beam under action of the end-forces was carried out by strength of materials relationships to obtain distributed normal and shear edge forces acting on the beam walls and to compute the corresponding average stresses. The beam column buckling was analyzed by a "designer handbook" type of closed form formulas provided in reference 13, based on the classical Eulerian approach.

Analysis of an individual stiffened plate extracted from the box beam included calculation of stress for the given edge-forces, equivalent Von Mises-Huber stress, and evaluation of several local buckling modes. These modes accounted for buckling of the sheet metal between the stringer and the plate edge, column buckling of the stringer with a cooperating strip of the sheet metal, buckling of the stringer web and flange, and flexural-twist buckling of the stringer elastically restrained by the sheet metal. A limited post-buckling analysis in the elastic

range was also included. The buckling analysis was carried out at the level characterized by reference 14 and it w>s coded in a program described in detail in ference 15. Detailed information on the side constraints on the design variables and on the geometrical proportions of the cross-sections is provided in the Appendix.

A general purpose program (ref. 16) based on the technique of usable-feasible directions was used for the design space search in the reference optimization and at every level of the three-level optimization.

Three-Level Optimization

To establish the reference results, the framework was first optimized without decomposition. Then, the multilevel optimization algorithm, variant 2 was applied to the structure decomposed as shown in Figures 3 and 4 showing the stiffened panels as daughters clustered in triplets (the fourth wall is symmetric) under a parent box beam. The beams, in turn, are daughters of the assembled structure.

The definition of the objective function, design variables, and constraints for each level in the decomposition is given in Table 1. As shown in the table, the top level optimization manipulates the beam extensional and bending stiffnesses through the cross-sectional area and bending moment of inertia. By coincidence, the area controls also the beam volume which contributes directly to the objective function.

At the middle level, the stiffnesses expressed by the area and moment of inertia become fixed parameters and the variables are the wall membrane stiffnesses controlled by the geometrical dimension variables. These variables, and consequently the membrane stiffnesses become fixed parameters at the bottom level at which the ultima*e detail dimensions are engaged as variables. The equality constraints arise between the parameters and variables. Owing to relative simplicity of the expressions involved, (see Appendix) these constraints are solved explicitly.

Examination of Table 1 in conjunction with the previous description of the analysis tools illustrates the point that dissimilar analysis may be used as needed at different places in a decomposition scheme.

The sensitivity analysis of behavior has been carried out by a single step forward finite difference technique. The optimum sensitivity analysis was based on the algorithm given in reference 5.

Results And Remarks On The Method Performance

Figures 5, 6, and 7 show a sample of results obtained when starting from with and without decomposition. To assure comparability of the results the starting points for both methods are the same. The normalized plots illustrate for each of the three different starting points, the objective function, a selected individual constraint, and a cumulative constraint containing the above individual constraint as they varied over the iterations in the optimization without decomposition and cycles in the three-level optimization. An iteration is defined in the optimization without decomposition as a usable-feasible directions iteration. A cycle is defined as one execution of the series of steps listed in the iterative procedure definition in the previous section.

The results verified that the multilevel algorithm was capable of finding a feasible design having an objective function close to and, in some cases, lower then the reference optimization without decomposition. As in reference 9, differences up to 72.1% were observed among the detailed design variables obtained by the two methods. However, these differences were no larger than those observed by comparing the designs obtained without decomposition starting from different initial design points. Therefore, these differences can be attributed to the problem non-convexity.

The volume of the data needed to describe the optimization results for the test structure is so voluminous that only a sample for one optimized case corresponding to Figure 6 is displayed in Table 2 to show dimensions of the optimized cross-sections.

The objective function minimum in the three-level optimization falling below the value obtained by the optimization without the decomposition was an unexpected result that occurred in several tests two of which are illustrated in Figures 5 and 6. Examination of the detailed numerical data suggested that these particular results were caused by both the different search path taken and by the larger number of constraints that the usable-feasible directions search algorithm had to process simultaneously in the optimization without decomposition. The algorithm implemented as described in reference 16 does not have a rigorous, Kuhn-Tucker based termination criterion. Instead, it terminates by a "practical" criterion of the diminishing returns in the value of the objective function. The criterion numerical tolerance was set the same for both methods. Proceeding from one constraint boundary to the next, as the usable-feasible directions algorithm does, the objective function reductions measured between the consecutive iterations tends to fall below that criterion prematurely, if there are many closely spaced constraint boundaries. In contrast, the multilevel scheme incorporates a piecewise linearization that inherently tends to produce significant increments of the objective function from one cycle to the next and tend to proceed further toward the theoretical constrained optimum than the reference method.

The graphs in Figures 5, 6, and 7 have a jagged appearance for both methods which is a characteristics of the usable-feasible directions search algorithm, amplified in the multilevel optimization by the extrapolation errors. However, these errors have never become excessive and the daughter substructure reactions to the changes in the parent design were effectively

predicted by the optimum sensitivity derivatives. In at least one case these predictions enabled the optimization at the middle level to remove the constraint violations at a bottom level substructure without any change to that substructure sizing.

Regarding the computational efficiency, it was not the purpose to demonstrate improvements of that efficiency and neither the reference nor the multilevel optimization procedures were honed for best computational performance in their implementations. The test case was computationally too small anyway to permit drawing conclusions as to the computational efficiency of any method. The only observation in this regard is that the amount of computational labor in one cycle tends to be less than in one iteration. Therefore, the total numbers of cycles and iterations should not be compared to evaluate the multilevel method efficiency. The main advantage of the multilevel method stems from its compatibility with the distributed computing technology.

Conclusions

An algorithm has been described for performing structural optimization by decomposing an optimization problem for an entire structure into a set of smaller subproblems. Each subproblem corresponds to a substructure in a general, substructuring analysis based on many levels of nested substructures. The optimization subproblems remain coupled by means of the optimum sensitivity analysis that generates derivatives of optimum solution with respect to constant parameters. These derivatives quantify the rate of the behavior change in a substructure optimized for constant inputs received from its governing, higher order substructure relative to the rate of change of these inputs. This quantification informs the optimizer modifying the higher order substructure about the effect its action will have on the subordinated substructures. The algorithm is intrinsically compatible with distributed computing because the subproblems are isolated and can be processed in parallel.

Testing on a framework structure made up of box beams that decomposes into a three-level pyramid of 13 subproblems showed that the optimization algorithm performed as expected or better. Such difficulties as have been encountered were not of the computational nature but were caused by complexity of data handling by the conventional FORTRAN means of named files and COMMON blocks. In fact, from the programmer's viewpoint this was a data-dominated problem and a conclusion was drawn that to proceed with a larger scale applications, with more levels and, particularly, with more engineering disciplines involved in the analysis would require systematic means of data handling such as those provided by modern data base management systems (e.g., ref. 17).

Comparison against a reference optimization without decomposition of the same structure demonstrated the validity and effectiveness of the multilevel optimization algorithm.

Satisfactory testing of the three-level optimization is seen as verifying an entirely general, multilevel algorithm because the formulation is recursive, and extension beyond three levels introduces no conceptually and qualitatively new elements into the decomposition scheme. The reported implementation is, therefore, regarded as a stage in the development of a multilevel, multidisciplinary optimization scheme applicable to engineering systems such as aircraft, due to the generality of the basic concepts underlying the algorithm.

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APPENDIX: Details of the Formulation

Additional or Redefined Nomenclature

A_z	Z-stiffener cross-sectional area
a	Z-stiffener leg cross-sectional area
b	X_2^M or X_5^M
I_y	Moment of inertia about the axis parallel to the webs
L	Move limit percentage
ℓ	Beam length
M	Bending moment
N	Resultant axial force
P	Axial force
Q	First moment of the area
R	Thickness-to-length ratio of the z-stiffeners legs
S	Section modulus
s	Slenderness (equation A10)
t	Smeared thickness
V	Shearing force
y	Centroidal distance to the extreme fiber
σ	Normal stress
τ	Shearing stress

Miscellaneous Information

The material was 6062-T62 aluminum defined by Table 48, reference (13). The box beams were assumed to be singly symmetric, allowing only one web to be modelled. The cross-sectional area of the z-stiffener was assumed to be a constant 10% ($r = 0.9$) of the smeared area of the flanges and 5% ($r = 0.95$) of the webs. The panels were assumed to be braced for local buckling every 24 inches.

Details of the Optimization Without Decomposition

Six design variables were used to model each z-stiffener. Six more design variables were used for the beam widths, X_2^M, and heights, X_5^M. The other beam dimensions were calculated by

$$A_z = X_1^B X_2^B + X_3^B X_5^B + X_2^B(X_8^B - X_1^B - X_3^B) \qquad \text{A1}$$

$$t = A_z/[b(1.0 - r)] \qquad \text{A2}$$

where $b = X_2^M$ for flange panels or X_5^M for web panels. Beam section properties were calculated from the dimensions. A finite element program was used to analyze the structure for displace-ments and end forces for the two independent load cases, 50,000 lbs. and 20 x 10^6 lb-in.

Four displacement constraints were computed, two for each load case:

$$D_j = \delta \text{ for each load case, } j = 1,2$$
$$c_j = 0.25 \text{ in.} \qquad \text{A3}$$

$$D_j = \theta \text{ for each load case, } j = 1,2$$
$$c_j = 0.005 \text{ rad.} \qquad \text{A4}$$

$$g_i = D_j/c_j - 1.0 \qquad i = 1,4 \qquad \text{A5}$$

For both loading cases, the stresses were evaluated at six points. Bending stresses were calculated at the bottom and top of both ends and shear stresses were evaluated at the neutral axis at both ends.

$$\sigma = \frac{P}{A} + \frac{My}{I} \qquad \text{A6}$$

$$\tau = \frac{VQ}{It} \qquad \text{A7}$$

When evaluating the stress constraints, a factor of safety was applied

$$D_j = 1.5\sigma$$
$$c_j = 26{,}000 \text{ psi for tension} \qquad \text{A8}$$

$$D_j = 1.5 \left| \tau \right|$$
$$c_j = 15{,}000 \text{ psi} \qquad \text{A9}$$

For compression, the allowable stress was reduced to prevent buckling. The compression allowable was computed by

$$s = Sl/I_y \qquad \text{A10}$$

For $s < 40$ $\quad c_j = 26{,}000$ psi

For $40 < s < 1810$ $\quad c_j = 1{,}000(27.4 - 0.22\sqrt{s})$psi

For $s < 1810$ $\quad c_j = 1810\ (18040)/s$ psi $\qquad$ A11

The function for the compression allowable was patterned after Table 4b of the ASCE specification for 6062-T6 aluminum in reference 13. The 36 stress constraints were

$$g_i = D_j/c_j - 1.0 \qquad i = 5,\ 40 \qquad \text{A12}$$

The end moments were converted to couples and summed with the axial force by

$$N_i = P_i + M_i/(X_1^M + X_3^M + X_5^M) \qquad \text{A13}$$

The resultant forces on both ends of the beam for both load cases were applied as four separate loadings. The shears and resultant forces for both flanges were applied to the webs, generating eight loadings. Thirteen constraints for each load case were evaluated to select the thirteen maximum values. This approach resulted in 117 constraints for all nine panels.

Additional geometric constraints were applied to the z-stiffener in the following forms

$$a_1 = X_1^B X_4^B,\ a_2 = X_3^B X_5^B,\ a_3 = X_2^B(X_6^B - X_1^B - X_3^B) \qquad \text{A14}$$

$$D_j = a_i$$
$$c_j = 0.1A_z \qquad \text{A15}$$

$$g_i = D_j/c_j - 1 \qquad i = 158,\ 184 \qquad \text{A16}$$

$$R_1 = X_1^B/X_4^B,\ R_2 = X_2^B/X_6^B,\ R_3 = X_3^B/X_5^B \qquad \text{A17}$$

$$D_j = R_j$$
$$c_j = 0.5 \qquad \text{A18}$$

$$g_i = D_j/c_j - 1.0 \qquad i = 185,\ 211 \qquad \text{A19}$$

The z-stiffener dimensions were also constrained to fit inside the box beam by

$$D_j = X_2^B,\ X_4^B,\ X_5^B \text{ for flanges}$$
$$c_j = (X_2^M - 2X_4^M)/4 \qquad \text{A20}$$

$$D_j = X_1^B,\ X_3^B,\ X_6^B \text{ for webs}$$
$$c_j = (X_2^M - 2X_4^M)/4 \qquad \text{A21}$$

$$D_j = X_1^B,\ X_3^B,\ X_6^B \text{ for flange}$$
$$c_j = X_5^M/4 \qquad \text{A22}$$

$$D_j = X_2^B,\ X_4^B,\ X_5^B \text{ for webs}$$
$$c_j = X_5^M/4 \qquad \text{A23}$$

$$g_i = D_j/c_j - 1.0 \qquad i = 212,\ 265 \qquad \text{A24}$$

Upper and lower bounds on the design variables were

$$5.0 < X_2^M,\ X_5^M < 100.0 \text{ for all cases} \qquad \text{A25}$$

$$0.0625 < X_1^B,\ X_2^B,\ X_3^B < 2.5$$
$$1.0 < X_4^B,\ X_5^B < 25.0 \qquad \text{for case 1}$$
$$1.125 < X_6^B < 25.0 \qquad \text{A26}$$

$$0.0625 < X_1^B,\ X_2^B,\ X_3^B < 1.0$$
$$1.0 < X_4^B,\ X_5^B < 10.0 \text{ for case 2 and 3}$$
$$1.125 < X_6^B < 10.0 \qquad \text{A27}$$

The objective function, F, was defined as the framework material volume,

$$F = \sum_{i=1}^{3} A_i l_i \qquad \text{A28}$$

<u>Details of the Multilevel Optimization</u>

Details of the bottom level formulation have been given in the preceding section. At the middle level L_i^M and U_i^M were defined for X_2^M and X_5^M by equation (A26). For X_1^M, X_3^M and X_4^M, L_i^M and U_i^M were calculated by replacing X_i^B with U_i^B and L_i^B in equation (A1) to calculate $A_{z,\ max}$ and $A_{z,\ min}$, respectively. Then

$$L_i^M = \frac{A_{z,\ min}}{b_{max}(1-r)} \qquad \text{A29}$$

$$U_i^M = \frac{A_{z,\ max}}{b_{min}(1-r)} \qquad \text{A30}$$

where $b_{min} = 5$ and $b_{max} = 100$ consistent with equation (A25).

The more restrictive of U^M and L^M or the move limits, calculated by

$$X^{U,L} = X^M(1 \pm L) \qquad \text{A31}$$

$$L = 0.075 \qquad \text{A32}$$

were chosen as the cycle's upper and lower bounds.

X_1^M, X_2^M, and X_3^M were used as design variables while X_4^M and X_5^M were calculated as dependent variables to satisfy equation (15) by

$$B_1 = 0.5X_2^M[(X_3^M)^2 - (X_1^M)^2]/A \qquad \text{A33}$$

$$B_2 = 0.5X_2^M[X_3^M - X_1^M]/A \qquad \text{A34}$$

$$B_3 = [A - X_2^M(X_1^M + X_3^M)]/12 + X_1^M X_2^M(B_2 + 1/2)^2 + X_2^M X_3^M(B_2 - 1/2)^2 + B_2^2(A - X_1^M X_2^M - X_2^M X_3^M) \qquad \text{A35}$$

$$B_4 = 2[X_1^M X_2^M(B_1 + X_1^M/2)(B_2 + 1/2) + X_2^M X_2^M(B_1 - X_3^M/2)(B_2 - 1/2) + B_1 B_2(A - X_1^M X_2^M - X_2^M X_3^M)] \qquad \text{A36}$$

$$B_5 = X_1^M X_2^M(B_1 + X_1^M/2)^2 + X_2^M X_3^M(B_1 - X_3^M/2)^2 + B_1^2(A - X_1^M X_2^M - X_2^M X_3^M) + X_2^M[(X_1^M)^3 + (X_3^M)^3]/12 - I \qquad \text{A37}$$

$$X_5^M = 0.5(-B_4 + \sqrt{B_4^2 - 4B_3B_5})/B_3 \qquad \text{A38}$$

$$X_4^M = 0.5(A - X_1^M X_2^M - X_2^M X_3^M)/X_5^M \qquad \text{A39}$$

At the top level, the more limits were defined as in equations A31 and A32.

Table 1. Quantities Defined for the Multilevel Test Case Optimization

	TOP LEVEL
OBJECTIVE:	The framework material volume
DESIGN VARIABLES:	A and I of the beams
CONSTRAINTS:	Displacements of the loaded corner and $\tilde{C}_e$ for the beams
	MIDDLE LEVEL
OBJECTIVE:	Cumulative constraint C representing the column buckling and $\tilde{C}_e$ for the walls.
DESIGN VARIABLES:	Wall membrane stiffness contributing to the beam axial and bending stiffnesses controlled through the dimensions shown in fig. 3, section A-A.
CONSTRAINTS:	Equality-beam cross-sectional area and moment of inertia
	BOTTOM LEVEL
OBJECTIVE:	Cumulative constraint C representing a set of stress and local buckling constraints of the wall.
DESIGN VARIABLES:	Cross-sectional dimensions shown in fig. 3 DETAIL B
CONSTRAINTS:	Inequality - minimum gages, geometrical proportions, and geometrical realizability.
	Equality - membrane stiffnesses for tension-compression and bending in of the wall in its own plane.

Table 2. REPRESENTATIVE RESULTS

	BEAM 1			BEAM 2			BEAM 3		
	INITIAL VALUES	SINGLE LEVEL FINAL VALUES	MULTI-LEVEL FINAL VALUES	INITIAL VALUES	SINGLE LEVEL FINAL VALUES	MULTI-LEVEL FINAL VALUES	INITIAL VALUES	SINGLE LEVEL FINAL VALUES	MULTI-LEVEL FINAL VALUES
$F(in^3)$	64347.5	57127.0	46060.5	64347.5	57127.0	46060.5	64347.5	57127.0	46060.5
$A(in^2)$	42.7	42.1	36.3	190.0	161.8	127.7	57.7	55.2	46.1
$I(in^4)$	3903.5	3894.0	3365.0	31088.1	41370.0	39550.0	5528.1	6169.0	4621.0
g_{max}	0.092	-0.097	0.044	0.092	-0.097	0.0438	0.0917	-0.097	0.044
C^M	-		0.006	-	-	-0.157	-	-	-0.067
$X_1^M(in)$	1.620		1.572	0.971		0.816	0.791		0.650
$X_2^M(in)$	10.0	9.997	8.524	35.0	34.34	31.32	20.0	19.96	18.57
$X_3^M(in)$	0.75		0.831	1.3		1.456	0.88		0.787
$X_4^M(in)$	0.423		0.354	1.727		0.677	0.518		0.420
$X_5^M(in)$	22.5	22.65	22.35	32.0	38.71	41.74	22.5	24.15	23.08
C^B	-	-	0.024	-	-	-0.182	-	-	-0.041
$X_1^B(in)$	0.1	0.1	0.089	0.8	0.845	0.851	0.3	0.297	0.383
$X_2^B(in)$	0.1	0.1	0.086	0.25	0.263	0.325	0.4	0.396	0.176
$X_3^B(in)$	0.4	0.398	0.358	0.8	0.845	0.851	0.3	0.297	0.383
$X_4^B(in)$	1.0	1.0	1.0	2.0	2.130	2.176	2.0	1.976	1.627
$X_5^B(in)$	1.0	1.0	1.004	2.0	2.130	2.176	2.0	1.976	1.627
$X_6^B(in)$	3.0	2.988	2.941	7.0	7.497	2.176	2.0	1.976	1.627
C^B	-	-	0.041	-	-	-0.227	-	-	-0.040
$X_1^B(in)$	0.3	0.299	0.35	0.5	0.495	0.285	0.175	0.174	0.072
$X_2^B(in)$	0.3	0.296	0.160	0.4	0.371	0.305	0.175	0.174	0.092
$X_3^B(in)$	0.3	0.299	0.321	0.5	0.495	0.165	0.5	0.491	0.491
$X_4^B(in)$	1.0	1.0	1.272	1.0	1.0	1.0	2.0	1.987	1.804
$X_5^B(in)$	1.0	1.0	1.240	1.0	1.0	1.721	2.0	1.963	1.667
$X_6^B(in)$	4.0	3.934	3.420	7.0	6.404	7.755	2.0	1.987	2.523
C^B	-	-	-0.110	-	-	-0.159	-	-	-0.144
$X_1^B(in)$	0.062	0.062	0.086	0.3	0.285	0.176	0.062	0.064	0.062
$X_2^B(in)$	0.1	0.099	0.063	0.625	0.513	0.172	0.062	0.064	0.069
$X_3^B(in)$	0.175	0.173	0.155	0.2	0.187	0.172	0.35	0.338	0.229
$X_4^B(in)$	2.0	1.986	1.545	2.0	1.856	1.940	1.0	1.0	1.0
$X_5^B(in)$	1.0	1.0	1.004	3.0	2.784	2.962	1.0	1.0	1.016
$X_6^B(in)$	2.0	1.977	1.950	3.0	2.343	2.851	3.0	2.938	3.131
Iterations		7	22-23		7	22-23		7	22-23

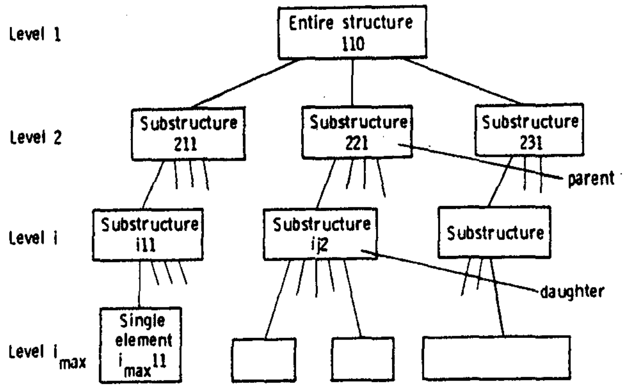

Fig. 1 Multilevel substructuring.

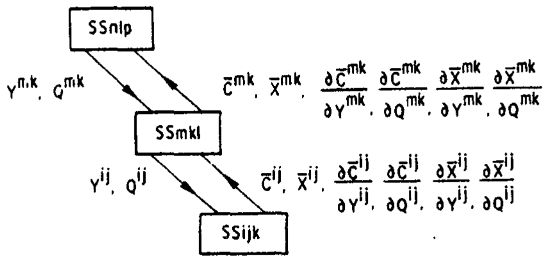

Fig. 2 Flow of information.

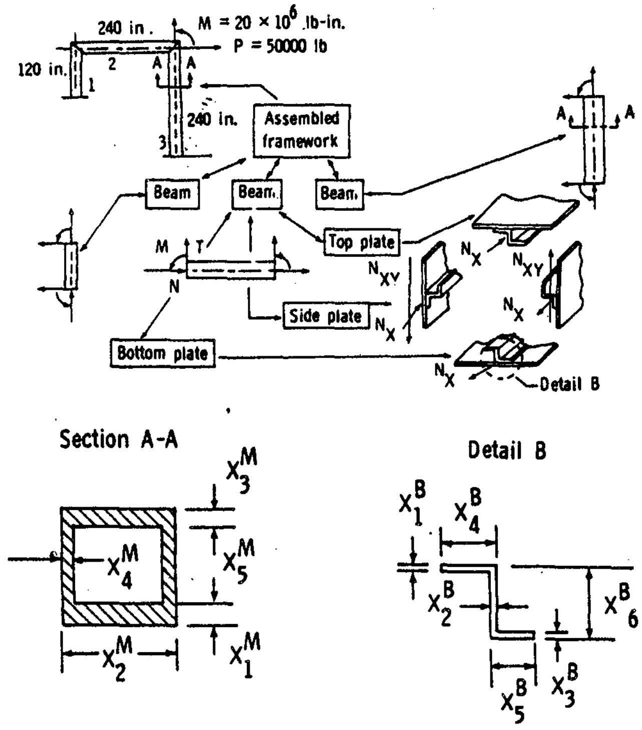

Fig. 3 A portal framework.

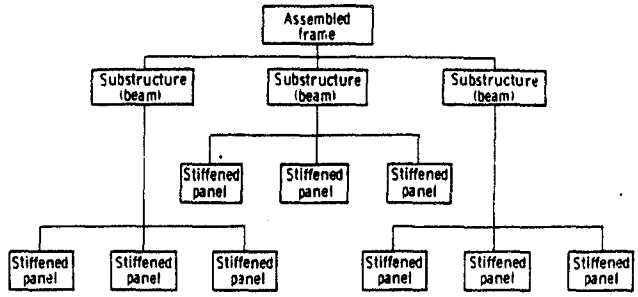

Fig. 4 Hierarchical decomposition of the framework structure shown in Fig. 3

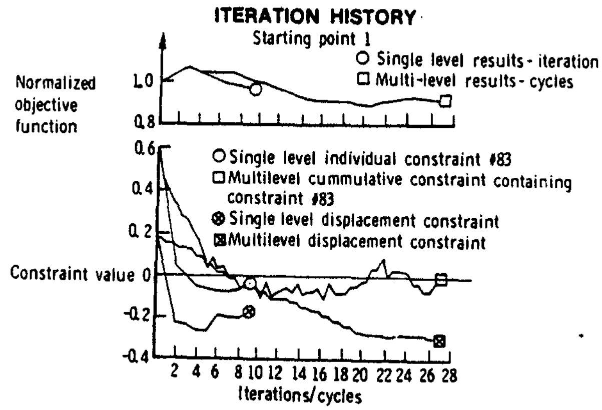

Fig. 5 Representative results.

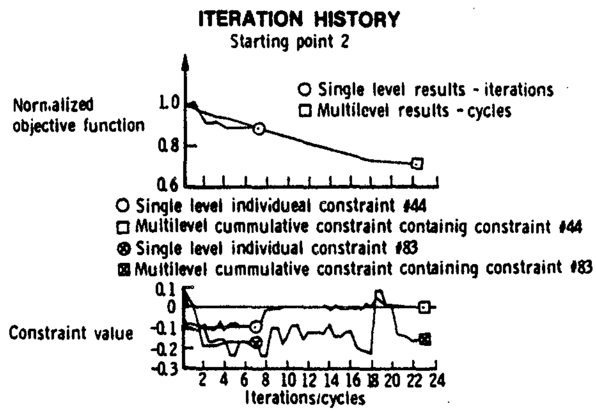

Fig. 6 Representative results.

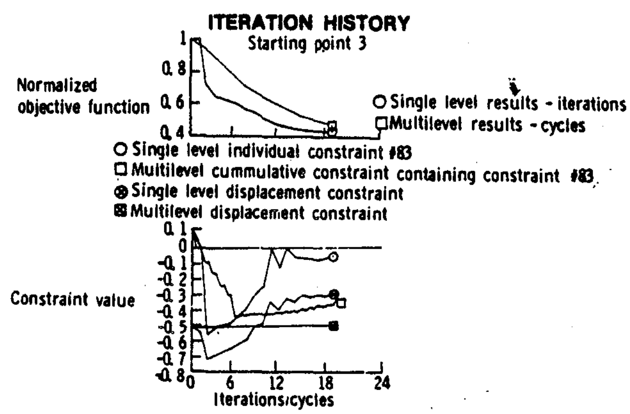

Fig. 7 Representative results.

1. Report No. NASA TM-87605	2. Government Accession No.	3. Recipient's Catalog No.
4. Title and Subtitle Structural Optimization by Generalized, Multilevel Decomposition		5. Report Date October 1985 6. Performing Organization Code 505-33-53
7. Author(s) J. Sobieszczanski-Sobieski; B. B. James; and M. F. Riley		8. Performing Organization Report No.
9. Performing Organization Name and Address NASA Langley Research Center Hampton, VA 23665		10. Work Unit No. 11. Contract or Grant No.
12. Sponsoring Agency Name and Address National Aeronautics and Space Administration Washington, DC 20546		13. Type of Report and Period Covered Technical Memorandum 14. Sponsoring Agency Code

15 Supplementary Notes

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16. Abstract

The developments toward a general multilevel optimization capability and results for a three-level structural optimization are described. The latter is considered a major stage in the method development because the addition of more levels beyond three does not introduce any new qualitative elements, so that a three-level implementation is qualitatively equivalent to a multilevel implementation.

The method partitions a structure into a number of substructuring levels where each substructure corresponds to a subsystem in the general case of an engineering system. The method is illustrated by a portal framework that decomposes into individual beams. Each beam is a box that can be further decomposed into stiffened plates. Consequently, substructuring for this example spans three different levels: the bottom level of finite elements representing the plates, an intermediate level of beams treated as substructures, and the top level for the assembled structure. This example is an extension of a case presented previously which was limited to two levels. Further extensions would add only more intermediate substructuring levels; therefore, the three-level case is qualitatively complete.

17. Key Words (Suggested by Author(s))	18. Distribution Statement
Structures, optimization	Unclassified - Unlimited Star Category 05

19 Security Classif. (of this report)	20. Security Classif. (of this page)	21. No. of Pages	22. Price
Unclassified	Unclassified	17	A02

For sale by the National Technical Information Service Springfield Virginia 22161